Relations between solitons and renormalized phonons in Fermi-Pasta-Ulam-Tsingou chians

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The Fermi–Pasta–Ulam-Tsingou (FPUT) model of nonlinearly coupled atoms is the simplest prototype for condensed matter systems. In FPUT chains and almost all the similar low-dimensional momentum-conserving nonlinear discrete lattices, thermal conductivity diverges with the length of lattices. The so-called anomalous energy transport has attracted increasing interest in recent two decades. The ubiquity of anomalous energy transport in low-dimensional lattices has been demonstrated experimentally. The underlying mechanism is still an open question. It is widely believed that the mechanism should be the same for anomalous energy transport and anomalous energy diffusion (superdiffusion). In the case of superdiffusion, numerical results reveal a pair of characteristic side peaks in the spatiotemporal correlation function of energy, momentum and other conserved quantities. They move outwards symmetrically at a constant supersonic velocity. This characteristic velocity is referred to as sound velocity and has been conventionally used to identify the energy carriers.

Solitons are spatially localized waves that can propagate over a long distance almost without deformation even after collision with phonons or solitons. The capability of exciting the supersonic solitons in the static FPUT-like lattices has been proved by a lot of theoretical and numerical approaches [1, 2]. Therefore, the supersonic solitons are naturally regarded as the energy carriers causing anomalous energy transport in FPUT-like lattices. However, there is still a debate on the existence of solitons at thermal equilibrium. The spatially localized structures of the particle displacements in FPUT chains at thermal equilibrium are regarded as solitons in Ref. [3]. But the similar spatially localized structures of the energy density are regarded as renormalized phonons in Ref. [4]. From a theoretical point of view, renormalized phonons are calculated by treating the nonlinear terms in the equations of motion with the mean field method. From a numerical perspective, renormalized phonons can be regarded as the temporal Fourier modes. Both solitons and renormalized phonons can well predict the sound velocity and there is still no method yet to distinguish them. Therefore, whether the energy carriers are solitons or renormalized phonons has generated a heated debate.

Recently, we have provided some new evidences showing that solitons can exist in the fluctuation environment [5]. We thus expect that supersonic solitons can exist at thermal equilibrium and lead to the anomalous energy diffusion in FPUT chains. To prove it, some observable unique features of solitons should be exploited. Because of thermal fluctuations, solitons cannot be detected directly at thermal equilibrium. However, it has been revealed that two kinds of solitons can be excited in FPUT-$\alpha\beta$ chains. They are referred to as rarefaction solitons and compressional solitons respectively. As functions of the relative displacement between the adjacent particles, solitons are bell-shaped. A rarefaction soliton center corresponds to a crest but a compressional soliton center corresponds to

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a trough. We have predicted that velocities of these two kinds of solitons are different at thermal equilibrium and the velocity of the low-frequency renormalized phonons is in between these two velocities [1, 2]. Based on this distinction in velocities, solitons in FPUT-$\alpha\beta$ chains at thermal equilibrium can be identified.

In this work, the spatiotemporal correlation functions of energy and momentum are numerically investigated in FPUT-$\alpha\beta$ chains at thermal equilibrium. There is a pair of side peaks in momentum correlation functions. As reported previously, this pair of side peaks move outwards symmetrically at the velocity of the low-frequency renormalized phonons. However, for energy correlation functions, besides a pair of primary side peaks, an additional pair of secondary side peaks are observed. The velocity of the primary side peaks coincides with the velocity of rarefaction solitons (the faster solitons). The velocity of the secondary side peaks approaches the velocity of compressional solitons (the slower solitons). To further identify solitons at thermal equilibrium and to clarify the relations between solitons and renormalized phonons, the evolution of the relative displacements of the adjacent particles are investigated. Crests move at the velocity of rarefaction solitons and troughs move at the velocity of compressional solitons as predicted. Moreover, these characteristic crests and troughs can be formed by the superposition of the low-frequency renormalized phonons (Fourier modes). We thus argue that the superposition of the low-frequency renormalized phonons forms solitons in FPUT chains at thermal equilibrium just like discrete breathers consisting of the superposition of the high-frequency renormalized phonons. These solitons (collective effects of renormalized phonons) cause the anomalous energy diffusion (transport) in FPUT chains.

Keywords: anomalous energy transport/diffusion; solitons; renormalized phonons; FPUT chains

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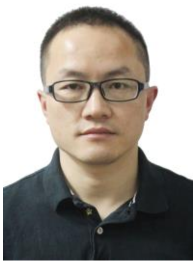
BIOGRAPHY



Yi Ming has completed his PhD from Physics Department, University of Science and Technology of China in 2006. He is now an associate professor at Anhui University.